Correlation of Laminar Boundary-Layer Quantities for Hypersonic Flows

William J. Cook*
Iowa State University, Ames, Iowa

Nomenclature

H = dimensionless static enthalpy, $(h-h_e)/h_e$
q = heat transfer rate per unit area
r = enthalpy recovery factor, $2H_r/(u_e^2/h_e)$
Re = Reynolds number, $xu_e\rho_e/\mu_e$
s = geometry parameter; cone, $s=1$, plate, $s=0$
St = Stanton number, $-q_w/\rho_e u_e(h_r-h_w)$
T = absolute temperature
u = x-component of velocity
β = u/u_e
δ = velocity boundary-layer thickness
$\rho\mu$ = density-viscosity product
ϕ = $\mu(\partial u/\partial y)(Re)^{1/2}/\rho_e u_e^2$
σ = effective Prandtl number

Subscripts

e = boundary-layer edge
r = recovery condition
w = wall

Theme

RESEARCH in high-speed flows includes studies of aerodynamic heating and boundary-layer behavior in gases and gas mixtures that simulate planetary atmospheres. The aim of the investigation reported herein was to correlate results for a group of typical gases for zero pressure gradient hypersonic laminar boundary-layer flows over simple configurations such as the sharp flat plate, the wedge, and the sharp cone. (A previous study by Marvin and Deiwert[1] has dealt primarily with correlation of stagnation-point heat transfer for several gases.) Such flows can readily be generated in test facilities such as the shock tunnel and the expansion tube in which testing typically involves models with cold surfaces relative to the recovery temperature. Hence, considerations here are directed toward the cold wall condition. In high Mach number flows real gas effects may play an important role since, due to pronounced dissipation effects, temperatures in the boundary layer can reach large values. Real gas effects in hypersonic flows may be present to varying degrees depending on the gas considered. As a result, precise universal correlations may not exist for all boundary-layer quantities, although the results in Ref. 1 for low values of dissipation suggest a small effect of property variation on heat transfer.

Contents

The method employed to solve the governing laminar boundary-layer equations is that of Crocco in which the usual equations in terms of x and y are transformed to equations in x and u. For thermochemical equilibrium in the flows considered, the transformed momentum and energy equations become respectively

$$\phi\phi_{\beta\beta}+3^s[\rho\mu/(\rho\mu)_e]\beta/2=0 \quad (1)$$

$$\phi\phi_\beta H_\beta=(u_e^2/h_e)\phi^2+\phi(\phi H_\beta/\sigma)_\beta \quad (2)$$

The boundary conditions are $\phi_\beta=0$, $H=H_w$ at $\beta=0$ (wall) and $\phi=0$, $H=0$ at $\beta=1$ (edge). The solutions $\phi(\beta)$ and $H(\beta)$ provide the basis for evaluation of the various boundary-layer quantities. The local Stanton number and skin friction coefficient expressions become

$$St(Re)^{1/2}=(3^{s/2})(\phi_{s=0}H_\beta)_w/\{\sigma_w[(ru_e^2/2h_e)-H_w]\} \quad (3)$$

$$c_f=\mu(\partial u/\partial y)/(\rho_e u_e^2/2)=2(3^{s/2})(\phi_{s=0})_w/(Re)^{1/2} \quad (4)$$

The factor $3^{s/2}$ permits application of the results for either $s=0$ or 1. In Eq. (3), r is determined by solving Eqs. (1) and (2) for H_r with $H_\beta(0)=0$ in Eq. (2). The distance normal to the wall is

$$y(Re)^{1/2}/x=\int_0^\beta(\mu/\mu_e\phi)\,d\lambda \quad (5)$$

Specification of the pressure, T_w, H_w, and u_e^2/h_e (Eckert number) for Eqs. (1) and (2), and consideration of various gases permits the influence on results of variations in properties among the gases to be assessed. Solutions have been obtained for Eckert numbers ranging from 10 to 70 and values of $H_w=0$ and -0.68 for $T_w=300$ K and a pressure of 1013 Pa (0.01 atm) employing real gas properties obtained mainly from Ref. 1. Corresponding Mach numbers range between about 5 and 13. The values of H_w correspond to $T_e/T_w=1$ and $T_e/T_w\simeq3$ respectively. Comparisons have

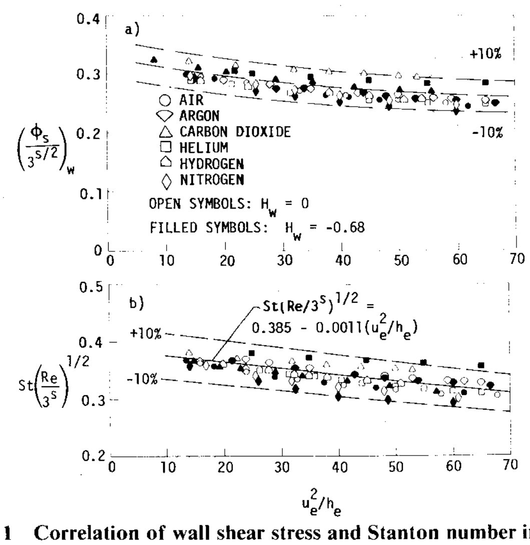

Fig. 1 Correlation of wall shear stress and Stanton number in terms of Eckert number.

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Index categories: Boundary Layers and Convective Heat Transfer—Laminar; Supersonic and Hypersonic Flow.

*Professor, Mechanical Engineering Department and Engineering Research Institue. Member AIAA.

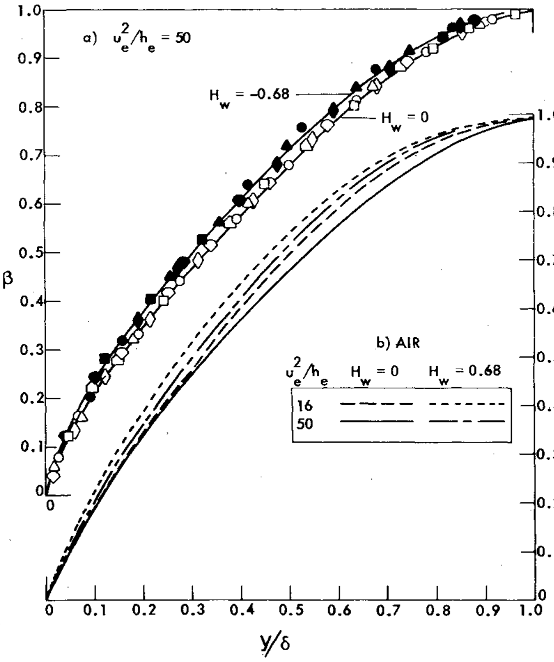

Fig. 2 Velocity profiles for six gases.

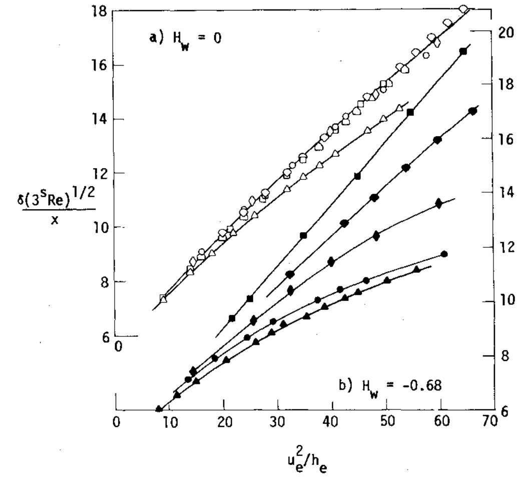

Fig. 3 Velocity boundary-layer thickness vs Eckert number for six gases.

been made between both the shear stress profiles $\phi(\beta)$ and the enthalpy profiles $H(\beta)$ obtained at fixed values of u_e^2/h_e and H_w for six gases. Little variation among the respective profiles was observed. Further, only small variations occured within the results for ϕ_w and the wall enthalpy gradient $(H_\beta)_w$ for the two values of H_w. This indicates a minor influence of property differences on these results at fixed Eckert numbers, a result consistent with the findings of Ref. 1. However, significant variations in shear stress and enthalpy quantities were observed with Eckert number. This is illustrated for wall shear stress in Fig. 1a for the six gases considered. The results for the various gases fall within ±10% of the fitted curve for both values of H_w. The Stanton number provides a means for correlating aerodynamic heating and requires the recovery factor be known for each gas considered. Computed values of r have been compared with those approximated by $\sigma_w^{1/2}$. Values of $r/\sigma_w^{1/2}$ were found to range from 0.92 to 1.01 for the six gases. Hence, for simplicity r was taken as $\sigma_w^{1/2}$. Figure 1b presents $St(Re/3^s)^{1/2}$ vs u_e^2/h^e computed from Eq. (3) with $r=\sigma_w^{1/2}$. It is seen that the results are closely grouped and that with the exception of helium at $H_w=-0.68$, the results fall within ±10% of a straight line fitted through the collective results. The present results illustrate that the main influence on shear stress and enthalpy quantities and related wall quantities is that of Eckert number. This is due to the relation between Eckert number and the viscous dissipation term. For the flows considered here the dissipation effects are dominant and are reflected in the large values of Eckert number. The dissipation effects present in this study are significantly larger than those considered in Ref. 1 in which it was found that small dissipation effects had little influence on results for the flows considered therein.

Knowledge of the Reynolds analogy factor 2 St/c_f permits the local skin friction to be predicted from local heat transfer measurements and vice versa. Reynolds analogy values determined with $r=\sigma_w^{1/2}$ for each of the six gases and for each value of H_w were found to be essentially independent of both Eckert number and H_w. The well-known Colburn analogy expresses the Reynolds analogy factor as $\sigma^{-2/3}$ for flows with constant Prandtl number. Comparisons made between values of $\sigma_w^{-2/3}$ and corresponding computed Reynolds analogy factors indicated excellent agreement (within 2%).

Equation (5) provides the basis for consideration of the boundary-layer velocity profiles for the various gases. Values of β vs y/δ (δ determined at $\beta=0.995$) which hold for either $s=0$ or $s=1$ have been computed for each of the six gases. Representative results are presented in Fig. 2a for $u_e^2/h_e=50$ and for both values of H_w. Curves drawn through the points for each value of H_w indicate a small influence of H_w on the velocity profiles. In addition, the velocity profiles are relatively insensitive to the gas considered. It was also found that the Eckert number has a relatively small influence on the velocity profiles. This is illustrated in Fig. 2b in which velocity profiles are shown for air at both values of H_w and for Eckert numbers of 16 and 50.

Momentum thickness results were found to correlate with Eckert number within ±10% for both values of H_w. The velocity and displacement thicknesses for the gases correlate well with Eckert number for the case $H_w=0$. However, for the case $H_w=-0.68$ these thickness quantities vary with both Eckert number and the gas considered. This is illustrated for the velocity thickness for $H_w=0$ and $H_w=-0.68$ in Figs. 3a and 3b respectively. The dependence on the gas considered for $H_w=-0.68$ is traced in the reference paper to large differences in viscosity between the gases that are present at the large values of H associated with the case $H_w=-0.68$.

Results of this study indicate that the Eckert number is in general a significant correlating variable for the flows considered. Those boundary-layer quantities, namely wall shear stress, wall enthalpy gradient, Stanton number, and momentum thickness, which depend mainly on the $\rho\mu$ product variation with enthalpy (and in the case of Stanton number, on Prandtl number) correlate with Eckert number within ±10%, with no important effect of H_w or pressure. Those quantities which depend explicitly on viscosity (the velocity and displacement thicknesses) do not correlate when large variations in enthalpy occur across the boundary layer. Although the correlations obtained should be applied only for the cold wall condition imposed and the range of H_w covered, it seems likely that similarities of the nature revealed here exist in other flows with large Eckert numbers.

Acknowledgment

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References

[1]Marvin, J. G. and Deiwert, G. S., "Convective Heat Transfer in Planetary Gases," NASA TR R-224, 1965.